INFO4990 Outline of Research Approach

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1 Introduction

This is an outline of the research approach for the project entitled "Game Theoretic Analysis of Internet Network Problems". This report will describe the proposed contributions, and methods of attaining those contributions. In summary, the proposed contributions are:

- Develop and analyse a theoretical model for inverse congestion games, and as a concrete application, "taxi routing".
- Explore additional variations of charging selfish users for the use of multicast, which is another application of the inverse congestion games.

2 Inverse Congestion Games

The proposed contributions in this area are:

- Develop a theoretical model for inverse congestion games, where congestion is either desired, or the edge cost functions are decreasing in the number of users. This differs from the standard congestion game model as defined in [10].
- Analyse the model's properties, including existence of Nash equilibria, the computational complexity of calculating a Nash equilibrium, and the price of anarchy [13, 7].
- Apply the results to a real-world example, namely "taxi routing", and use real-world data to verify the theoretical results experimentally.
- Investigate the relationship between the inverse congestion game model and the selfish multicast model.

A theoretical model will be created by extension of the Roughgarden traffic-based model [12], such that the simple cases of this will be able to be compared with the examples in Pigou's Paradox and Braess' Paradox [13]. The notions of edge costs will be borrowed from the multicast routing models, such as in [5] and [8]. Two different models, one based on splittable flow and the other on unsplittable flow, will be analysed, in the spirit of [3]. An additional modification, namely the addition of edge constraints, will be introduced into the model, by analogy with [8]. Simple examples should suffice to capture the essence of the complexity of the model [11], and in particular, we will confine ourselves to linear cost functions and a restricted network topology.

The model's properties will be analysed by way of proof. The existence of Nash equilibria in the "taxi routing" model is likely to be a non-constructive existence proof, and could make use of Kakutani's fixed point theorem [6] as in Nash's original proof of the existence of mixed equilibria [9], or it could be proved by way of using a potential function [10]. The mode should be able to be formulated as a linear program, and convex optimisation and results about feasibility could be used to show existence [12]. As for computational complexity, the project will attempt to prove analogous results for cases with existing polynomial time algorithms, such as the symmetric network case [4]. The linear program reduction techniques in [8] will be applied in an attempt to avoid exponentially-sized LPs.

We will attempt to prove analogous bounds for the price of anarchy and the bicriteria bound [13, 12] by using a similar technique: the upper bound can be found by expressing the Nash flow in term of the optimal flow via the marginal cost function, and the bound can be made tight by showing a simple example [11]. The technique in [2], where the delay encountered by each agent is compared to the delay it would encounter if it changed to the optimal route, and the bounds are combined in a weighted fashion to obtain a relation between the total delays of the Nash flow and the optimal flow, may be of use.

It is understood that NICTA possesses some real-world data regarding journeys in taxis, although its applicability is yet to be determined. A simulation would be useful not only for verification of the theoretical results, but also to provide ideas about properties that can be proved theoretically. The model will be created as defined in the theoretical results, and extensions that are too complicated to be modelled mathematically may be able to be integrated into the simulation. As for the tools, MATLAB is likely to be used, along with CPLEX, for computing the price of anarchy and properties of equilibria.

Finally, we will attempt to link the new theoretical insights provided by the "taxi routing" model back to the multicast routing model via a comparison of the edge cost functions, which are similar in both cases. This link is useful in order to apply the charging algorithms to the taxi routing case. The lack of a encodable and replicable information flow, as defined in [1], will be investigated.

3 Charging for Multicast

The proposed contributions in this area are:

- Investigate the effects of applying a tax to the case where the receivers, instead of the flows, are charged for any transmission they receive.
- Investigate the effect of demanding different properties from the cost-sharing mechanism, including strategyproofness, no positive transfers, voluntary participation, consumer sovereignty, budget-balance and efficiency (not all can be satisfied at once).

Taxes were investigated in [8] and were found to be useful in modifying the selfish behaviour to produce an optimal result, even after the taxes were refunded. This result was studied in light of charging the *flow*, whereas we will seek to study the result in light of charging the *receivers* of the content, as in [5]. [5] detailed a list of desirable criteria for cost-sharing mechanisms, but only two mechanisms were studied; a different selection of criteria can be studied by way of the communication complexity, and in particular, the novel algebraic approach used in that paper.

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