INFO4990 Database Search Results for the project "Game Theoretic Analysis of Internet network problems"

Enoch Lau
SID 200415765

2 April 2007

Contents

1 Project description

In a network, greedy, independent agents aim to minimise their own personal cost (such as travel time between source and destination) without regard to wider, societal impacts of their behaviour. The inefficiency due to this behaviour can be studied through such measures as the price of anarchy, which is the ratio of the cost of the worst-case Nash equilibrium to that of the optimal flow. One application of this game theoretic analysis is in allowing network operators to charge users equitably and profitably for multicast traffic sent through their networks, because multicast traffic along a link cannot be simply attributed to one particular user. The aim of the project is to extend and modify existing models of multicast pricing, to improve the resultant societal cost even with greedy, independent agents and their applicability to real-world multicast uses. Theoretical properties, such as time and network overhead complexity, will be examined to determine the tractability of the models.

2 Paper summaries

2.1 Algorithms, Games and the Internet [35]

This paper surveys the current open problems that lie at the intersection of the Internet, game theory and economics. The author notes that the Internet is particularly open to being analysed in terms of game theory due to its participants that have diverse economic interests. Open problems (at the time of writing of the paper, which may have been resolved by now), include computing a mixed Nash equilibrium in polynomial time, finding an optimum solution to the multi-commodity flow problem with coalitions of allied players, and an estimation of the price of anarchy of the structure of the Internet. These problems are the open problems in this particular area of research, and this paper helps set the context for my project, as well as provide useful definitions and linkages between the various incremental results.

2.2 A Class of Games Possessing Pure-Strategy Nash Equilibria [37]

This work is a seminal work for the existence of pure Nash equilibria for non-cooperative games. The author begins with defining a congestion game, and provides examples. The work is interesting because it sets the scene for the frustration that later authors, such as in [10], have with the non-constructive nature of many Nash equilibrium existence proofs.

2.3 The Complexity of Pure Nash Equilibria [10]

The paper begins by noting that the Nash equilibrium existence theorem is non-constructive, and Nash equilibria tend to be difficult to find in practice. The paper describes a certain class of games that converge to a Nash equilibrium, when given a suitable potential function. However, it warns against exact potential function games, as they are isomorphic to congestion games, for which there is no known polynomial-time algorithm. This paper is relevant to the project, because it is useful in understanding other papers that use such potential functions in their proofs, such as [6], and in general, for useful analysis to be able to be performed on a system, it must first be shown to converge.

2.4 How Bad is Selfish Routing? [42]

The work demonstrates that, for a restricted case (where edges have a linear cost function), the price of anarchy is at most $\frac{4}{3}$, and for the general case, the cost of the Nash equilibrium is at most that of an optimal flow forced to route twice as much traffic. For both of these cases, it is assumed that each agent controls a negligible amount of flow; that is, the results are the limit of the multicast routing problem as the number of users approaches infinity. However, this assumption is unrealistic, and the work then progresses to prove analogous results for the finite case. This work is relevant as it is the seminal work on the application of the price of anarchy to study network flows, and one can garner the general techniques and concepts from it. However, it does not have direct applicability to the multicast pricing problem, because the edge costs are taken to be a non-decreasing function and the graphs contain only single source-destination pairs.

2.5 Min-Cost Multicast of Selfish Information Flows [24]

This work analyses optimal multicast flows in two cases: where edges do not have capacity limits and where edges do have limits. In the first case, the author shows that the Shapley value cannot enforce optimal multicast flows. The author then formulates a pair of primal and dual linear programs, and shows that the Nash equilibrium corresponds to an optimal multicast flow and achieves a balanced budget. In the second case, because optimal flow is not enforced, the author proposes a system of taxes that are later refunded to achieve the desired result. The author then discusses methods of optimising the exponentially large linear programs, by using Lagrange relaxation and the subgradient method. This paper is interesting because it achieves the desired outcome (Nash equilibrium with optimal flow) using linear programming, which distinguishes it from all the other papers so far. The idea of refundable taxes adds another layer of control that can be experimented with in other models that have not yet had taxes applied. Furthermore, the linear programming reduction techniques are useful in the practical realisation of any large theoretical models.

2.6 Non-Cooperative Multicast and Facility Location Games [6]

The work considers the multicast model with selfish players, but the novelty is that it considers the problem using best-response dynamics, whereby the players are introduced to the system one by one. Assuming that the cost of an edge is evenly split amongst its users, the authors derive upper limits for the price of anarchy for two cases: integral (unsplittable) and fractional (splittable) flow. The authors resort to the technique of level trees to obtain the upper bounds, and use a reduction from 3SAT to obtain NP-hardness. This paper is relevant because it establishes the societal cost with assumptions that are relevant to multicast routing, and it sets the backdrop of further improvements in the bound or the motivation to introduce economic incentives. An interesting open question noted is the existence of the Nash equilibrium for the weighted integral multicast game.

2.7 Sharing the Cost of Multicast Transmissions [12]

This paper discusses two decentralised cost-sharing mechanisms for multicast transmissions, namely marginal cost and the Shapley value. Economic considerations, such as No Positive Transfers (NPT), Voluntary Participation (VP), Consumer Sovereignty (CS), budget-balance and efficiency, together with strategyproofness (where there is a disincentive for an agent to lie about its utility function), restrict consideration to these two mechanisms. However, because of the quadratic number of messages that need to be sent, the authors state that the Shapley value is infeasible as a cost-sharing mechanism due to its overhead. Furthermore, the authors examine a multicast tree that emphasises optimality over stability and find that it is NP-hard to approximate within a constant ratio. This work is noteworthy because of the constraints it sets upon cost-sharing mechanisms (economic factors and network complexity) to arrive at a recommendation for the marginal cost method. It would be interesting to see the result of the selected relaxation of these factors. A potential research question arising from this paper is whether the quadratic network complexity of the Shapley value is truly a limiting factor, given the high bandwidth of today's networks. Furthermore, the algebraic technique in the proof of the lower bound for the Shapley value is applicable for similar proofs.

3 List of references

[1] M. Adler and D. Rubenstein. Pricing multicasting in more flexible network models. *ACM Trans. Algorithms*, 1(1):48–73, 2005.

[2] E. Anshelevich, A. Dasgupta, J. Kleinberg, E. Tardos, T. Wexler, and T. Roughgarden. The price of stability for network design with fair cost allocation. In *FOCS '04: Proceedings of the 45th Annual IEEE Symposium on Foundations of Computer Science (FOCS'04)*, pages 295–304, Washington, DC, USA, 2004. IEEE Computer Society.

[3] B. Awerbuch, Y. Azar, and A. Epstein. The price of routing unsplittable flow. In *Proceedings of the thirty-seventh annual ACM symposium on Theory of computing*, pages 57–66, New York, NY, USA, 2005. ACM Press.

[4] V. Bilè, C. D. Francescomarino, M. Flammini, and G. Melideo. Sharing the cost of multicast transmissions in wireless networks. In *Proceedings of the sixteenth annual ACM symposium on Parallelism in algorithms and architectures*, pages 180–187, New York, NY, USA, 2004. ACM Press.

[5] M. Bläser and B. Manthey. Budget balanced mechanisms for the multicast pricing problem with rates. In *Proceedings of the 4th ACM conference on Electronic commerce*, pages 194–195, New York, NY, USA, 2003. ACM Press.

[6] C. Chekuri, J. Chuzhoy, L. Lewin-Eytan, J. S. Naor, and A. Orda. Non-cooperative multicast and facility location games. In *Proceedings of the 7th ACM conference on Electronic commerce*, pages 72–81, New York, NY, USA, 2006. ACM Press.

[7] G. Christodoulou and E. Koutsoupias. The price of anarchy of finite congestion games. In *Proceedings of the thirty-seventh annual ACM symposium on Theory of computing*, pages 67–73, New York, NY, USA, 2005. ACM Press.

[8] R. Cole, Y. Dodis, and T. Roughgarden. How much can taxes help selfish routing? In *Proceedings of the 4th ACM conference on Electronic commerce*, pages 98–107, New York, NY, USA, 2003. ACM Press.

[9] B. M. Edwards, L. A. Giuliano, and B. R. Wright. *Interdomain multicast routing: practical Juniper Networks and Cisco Systems solutions*. Addison-Wesley Professional, 2002.

[10] A. Fabrikant, C. Papadimitriou, and K. Talwar. The complexity of pure Nash equilibria. In *Proceedings of the thirty-sixth annual ACM Symposium on Theory of Computing*, pages 604–612, New York, NY, USA, 2004. ACM Press.

[11] J. Feigenbaum, A. Krishnamurthy, R. Sami, and S. Shenker. Approximation and collusion in multicast cost sharing. In *Proceedings of the 3rd ACM conference on Electronic Commerce*, pages 253–255, New York, NY, USA, 2001. ACM Press.

[12] J. Feigenbaum, C. Papadimitriou, and S. Shenker. Sharing the cost of multicast transmissions. *Journal of Computer and System Sciences*, 63:21–41, 2001.

[13] L. Fleischer, K. Jain, and M. Mahdian. Tolls for heterogeneous selfish users in multicommodity networks and generalized congestion games. In *Proceedings of the 45th Annual IEEE Symposium on Foundations of Computer Science*, pages 277–285, Washington, DC, USA, 2004. IEEE Computer Society.

[14] M. Gairing, T. Lücking, M. Mavronicolas, and B. Monien. The price of anarchy for polynomial social cost. *Theor. Comput. Sci.*, 369(1-3):116–135, 2006.

[15] S. Herzog, S. Shenker, and D. Estrin. Sharing the ”cost” of multicast trees: an axiomatic analysis. In *Proceedings of the conference on Applications, technologies, architectures, and protocols for computer communication*, pages 315–327, New York, NY, USA, 1995. ACM Press.

[16] K. Jain and V. Vazirani. Applications of approximation algorithms to cooperative games. In *Proceedings of the thirty-third annual ACM symposium on Theory of computing*, pages 364–372, New York, NY, USA, 2001. ACM Press.

[17] K. Jain and V. V. Vazirani. Approximation algorithms for metric facility location and k-median problems using the primal-dual schema and lagrangian relaxation. *J. ACM*, 48(2):274–296, 2001.

[18] G. Karakostas and S. G. Kolliopoulos. Edge pricing of multicommodity networks for heterogeneous selfish users. In *Proceedings of the 45th Annual IEEE Symposium on Foundations of Computer Science*, pages 268–276, Washington, DC, USA, 2004. IEEE Computer Society.

[19] I. Keidar, R. Melamed, and A. Orda. EquiCast: scalable multicast with selfish users. In *Proceedings of the twenty-fifth annual ACM symposium on Principles of distributed computing*, pages 63–71, New York, NY, USA, 2006. ACM Press.

[20] F. Kelly. *Mathematics Unlimited - 2001 and Beyond*, chapter Mathematical modelling of the Internet, pages 685–702. Springer-Verlag, 2001.

[21] Y. A. Korilis, A. A. Lazar, and A. Orda. Avoiding the Braess paradox in non-cooperative networks. *J. Appl. Probab*, 36(1):211–222, 1999.

[22] A. A. Lazar, A. Orda, and D. E. Pendarakis. Virtual path bandwidth allocation in multiuser networks. *IEEE/ACM Trans. Netw.*, 5(6):861–871, 1997.

[23] A. Legout, J. Nonnenmacher, and E. W. Biersack. Bandwidth-allocation policies for unicast and multicast flows. *IEEE/ACM Trans. Netw.*, 9(4):464–478, 2001.

[24] Z. Li. Min-cost multicast of selfish information flows. In *Proceedings of IEEE INFOCOM 2007*, 2007.

[25] Z. Li, B. Li, and L. C. Lau. On achieving maximum multicast throughput in undirected networks. *IEEE/ACM Trans. Netw.*, 14(SI):2467–2485, 2006.

[26] D. S. Lun, N. Ratnakar, M. Médard, R. Koetter, D. R. Karger, T. Ho, E. Ahmed, and F. Zhao. Minimum-cost multicast over coded packet networks. *IEEE/ACM Trans. Netw.*, 14(SI):2608–2623, 2006.

[27] M. Mavronicolas and P. Spirakis. The price of selfish routing. In *Proceedings of the thirty-third annual ACM symposium on Theory of computing*, pages 510–519, New York, NY, USA, 2001. ACM Press.

[28] A. Mehta, S. Shenker, and V. V. Vazirani. Profit-maximizing multicast pricing by approximating fixed points. In *Proceedings of the 4th ACM conference on Electronic commerce*, pages 218–219, New York, NY, USA, 2003. ACM Press.

[29] I. Milchtaich. Congestion games with player-specific payoff functions. *Games and Economic Behavior*, 13(1):111–124, 1996.

[30] D. Monderer and L. S. Shapley. Potential games. *Games and Economic Behavior*, 14(1):124–143, 1996.

[31] H. Moulin and S. Shenker. Strategyproof sharing of submodular costs: budget balance versus efficiency. *Economic Theory*, 18(3):511–533, 2001.

[32] J. E. Nash Jr. Equilibrium points in n-person games. *Proc. Nat. Acad. Sci. US*, 36:48–49, 1950.

[33] G. L. Nemhauser and L. A. Wolsey. Best algorithms for approximating the maximum of a submodular set function. *Mathematics of Operations Research*, 3(3):177–188, 1978.

[34] M. J. Osborne. *A Course in Game Theory*. MIT Press, Cambridge, Massachusetts, 1994.

[35] C. H. Papadimitriou. Algorithms, games, and the internet. In *Proceedings of the thirty-third annual ACM Symposium on Theory of Computing*, pages 749–753, New York, NY, USA, 2001. ACM Press.

[36] C. H. Papadimitriou, A. A. Schäffer, and M. Yannakakis. On the complexity of local search. In *Proceedings of the twenty-second annual ACM symposium on Theory of computing*, pages 438–445, New York, NY, USA, 1990. ACM Press.

[37] R. W. Rosenthal. A class of games possessing pure-strategy Nash equilibria. *International Journal of Game Theory*, 2:65–67, 1973.

[38] M. Rothkopf, A. Pekec, and R. Harstad. Computationally manageable combinatorial auctions. *Management Science*, 44(8):1131–1147, 1998.

[39] T. Roughgarden. Designing networks for selfish users is hard. In *Proceedings of the 42nd IEEE symposium on Foundations of Computer Science*, page 472, Washington, DC, USA, 2001. IEEE Computer Society.

[40] T. Roughgarden. The price of anarchy is independent of the network topology. In *Proceedings of the thiry-fourth annual ACM symposium on Theory of computing*, pages 428–437, New York, NY, USA, 2002. ACM Press.

[41] T. Roughgarden. *Selfish Routing and the Price of Anarchy*. MIT Press, Cambridge, Massachusetts, 2005.

[42] T. Roughgarden and E. Tardos. How bad is selfish routing? *Journal of the ACM*, 49(2):236–259, 2002.

[43] T. Roughgarden and E. Tardos. Bounding the inefficiency of equilibria in nonatomic congestion games. *Games and Economic Behavior*, 47(2):389–403, 2004.

[44] A. S. Schulz and N. S. Moses. On the performance of user equilibria in traffic networks. In *Proceedings of the fourteenth annual ACM-SIAM symposium on Discrete algorithms*, pages 86–87, Philadelphia, PA, USA, 2003. Society for Industrial and Applied Mathematics.

[45] L. S. Shapley. A value for n-person games. *Contributions to the Theory of Games*, 2(28):307–317, 1953.

[46] A. Vetta. Nash equilibria in competitive societies, with applications to facility location, traffic routing and auctions. In *Proceedings of the 43rd Symposium on Foundations of Computer Science*, page 416, Washington, DC, USA, 2002. IEEE Computer Society.

[47] B. Vöcking. Congestion games: Optimization in competition. In H. Broersma, S. Dantchev, M. Johnson, and S. Szeider, editors, *Algorithms and Complexity in Durham 2006, Proceedings of the second ACiD Workshop*, volume 7 of *Texts in Algorithmics*, pages 9–20. College Publications London, 2005.

[48] W. Wang, X.-Y. Li, and Y. Wang. Truthful multicast routing in selfish wireless networks. In *Proceedings of the 10th annual international conference on Mobile computing and networking*, pages 245–259, New York, NY, USA, 2004. ACM Press.

[49] A. Zelikovsky. A series of approximation algorithms for the acyclic directed Steiner tree problem. *Algorithmica*, 18(1):99–110, 1997.

[50] J. Zhao, P. Zhu, X. Lu, and F. Zhao. A practical pricing model of inter-domain multicasting based on game theory. In *Proceedings of 5th ACM SIGCOMM workshop on Network and system support for games*, page 40, New York, NY, USA, 2006. ACM Press.