Game Theoretic Analysis of Network Problems

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What is this project about?

How much worse does a network perform when we allow users to route their traffic in a selfish manner?

Context

GAME THEORY
+
THEORETICAL CS

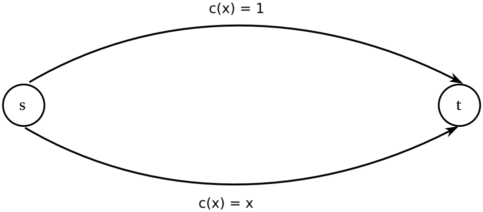
Research themes:

- Economic notions of game theory
- Theoretical computer science

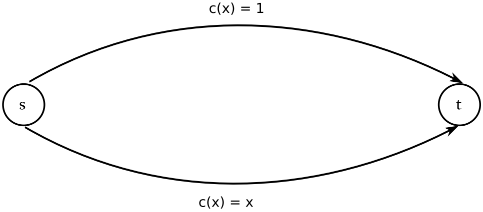
Putting the two together:

- The Internet is fertile ground for such analysis
- Use mathematical tools to reason about the behaviour of systems of users

Pigou's example



Pigou's example



- Optimal routing: half the traffic on each route
- Selfish routing: everyone goes on the bottom link

Pigou's example

Selfish behaviour need not produce a socially optimal outcome.

Network Model

- Directed graph with source-destination pairs called commodities
- Each commodity routes a certain amount of traffic, which can be carried over multiple paths
- Edges have a cost function, which may depend on the amount of flow on the edge (*congestion game*)
- Each user controls a negligible amount of flow
- Economics interpretation in addition to computer science

Nash Equilibrium

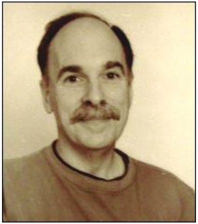
- Formalises what we mean by selfish behaviour in a game
- No user can gain by changing strategies unilaterally
- Stable outcome
- Pure Nash equilibrium: choose one strategy
- Mixed Nash equilibrium: choose from a set of strategies with a probability distribution

How to measure the effects of selfish users?

- Price of stability: Ratio of cost of *best* Nash equilibrium to cost of optimal routing
- Price of anarchy: Ratio of cost of *worst* Nash equilibrium to cost of optimal routing

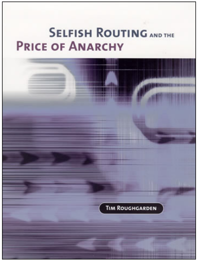
Classical Results

- Nash: every finite game has a mixed equilibrium (1950)
- Rosenthal: every congestion game has pure equilibria (1973)



Network Games

- Roughgarden and Tardos: initiated the price of anarchy in non-atomic network games in 2002
- Nash equilibrium at most 33% worse than optimal routing with linear edge cost functions
- Nash flow is no worse than an optimal flow forced to route twice as much traffic
- Network topology is irrelevant
- Simplest cases show worst behaviour



Extensions to the Model

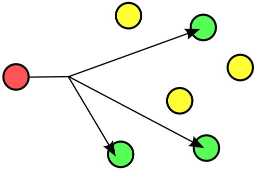
Changed assumptions about the network:

- Non-atomic vs. atomic congestion games
- Splittable vs. unsplittable flow
- ϵ-approximate Nash equilibria
- Relaxed assumptions on edge cost functions

Changed assumptions about the users:

- Malicious users
- Oblivious users

What is Multicast?



- Data is sent to multiple recipients, but is sent down each link only once
- Having multiple users on an edge is now good
    - Congestion games turned on its head
    - Use similar ideas though

Price of Anarchy of Multicast

- Two traditional economics-based edge-cost sharing mechanisms:
  - Shapley value
  - Marginal cost function
- Economic incentives can be used to encourage optimal behaviour, e.g. taxes

Reciprocal/Inverse Congestion Games

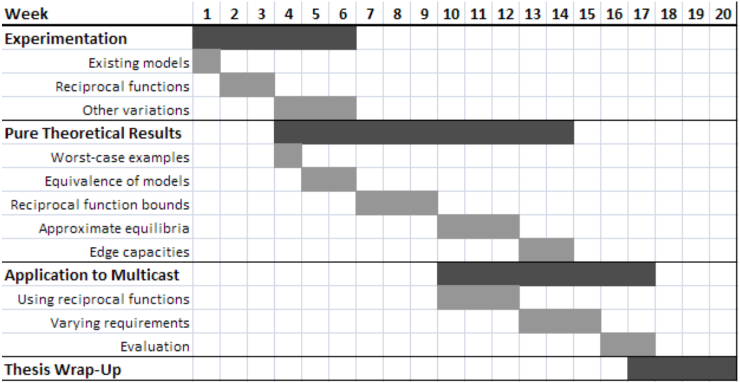
- To date, no one has examined decreasing edge-cost functions except in the restricted case of certain economics-based mechanisms
- Natural interpretations of decreasing edge-cost functions
- Produce equivalent results to literature, e.g.:
    - Price of stability and anarchy
    - Pure and mixed equilibria
    - Splittable and unsplittable flow
    - Edge capacities
    - Algorithms to compute equilibria
- Mixed increasing and decreasing functions

Applications to Multicast Routing

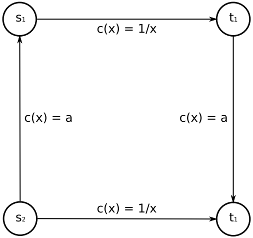
Reapply the generalisation to multicast:

- Generalisation of former charging models could result in a fairer charging scheme
- Computational and network complexity for the generalised model

Stages of the Project



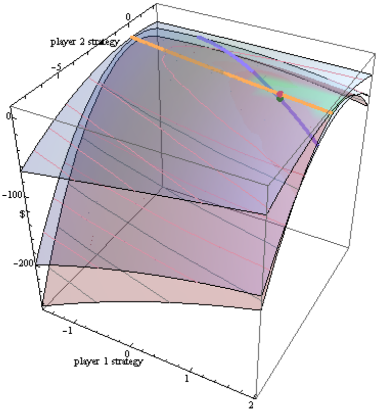
Exploration of Different Networks



- Experimentation is not a key component nor written about in theoretical computer science, but useful to get ideas
- Find small networks that capture the essence of the problem
- Systematic exploration of classes of networks:
  - Sparse/dense networks
  - Statistical distributions over traffic rates

Software

- AMPL/CPLEX: Optimiser for mathematical programming models expressed in algebraic form
- GAMUT/Gambit: Generates randomised games and finds Nash equilibria in restricted cases
- Mathematica: Visualisation of strategy spaces



Stage 1 Risks

- Cannot find generalisations: focus on restricted classes of networks, or make more assumptions about the edge-cost functions
- Software does not directly solve problem due to violated assumptions: pre or post processing required

In general, risks will be identified by comparison with timeline, and fallbacks initiated if necessary

Proof Techniques

- Linear and convex optimisation
- Reformulation of programs to reduce exponential size
- Use of marginal cost function to relate optimal and Nash flows
- Augmentation of optimal flows to attain Nash flows
- Lower bounds proved by simple examples
- Potential functions

Stage 2 Risks

- No guarantee of convexity: many simple mathematical tools such as convex optimisation with KKT conditions ruled out; novel approach in one paper to map to a different type of user equilibria
- No generalisation possible: find price of anarchy bounds in restricted network cases
- Risks minimised by adopting proofs in literature as a template
- Fallback: report numerical analyses

Stage 3 Risks

- Not core to the project, but good to demonstrate application
- Main risk is that the generalisation does not match reality
- Evaluation: analysis of computational and network complexity; re-useable approach outlined in one paper

Summary

- Selfish users in a network can cause suboptimal global outcomes
- We turn this on its head and examine what happens when we treat congestion as a good thing
- Generalise previous work into inverse congestion functions
- Application in fairer multicast pricing

Acknowledgements

I thank Dr Viglas for his supervision thus far.

Images:

- Roughgarden's book: http://mitpress.mit.edu/catalog/item/default.asp?tid=10339&ttype=2
- Multicast: http://en.wikipedia.org/wiki/Image:Multicast.svg
- John Nash: http://en.wikipedia.org/wiki/Image:John_f_nash_20061102_3.jpg
- Robert Rosenthal: http://www.ams.org/featurecolumn/archive/rationality.html

$\exists\, p \in$ audience s.t. p has some $q \in \{$ set of all questions $\}$?