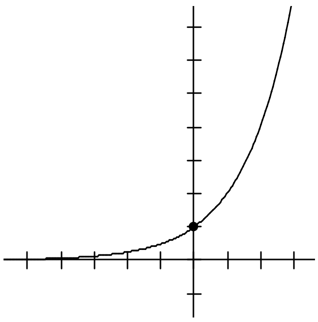
Exponential and Logarithmic Functions (Mathematics)

Exponential Functions



Differentiation

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{a^{x+h}-a^x}{h}\\ &= \lim_{h\to 0}\frac{a^x a^h - a^x}{h}\\ &= a^x \lim_{h\to 0}\frac{a^h-1}{h}\\ &= ka^x\end{aligned}$$

If $y = a^x$,
$$\frac{dy}{dx} = \log_e a \cdot a^x \text{ or } \ln a \cdot a^x$$

For $y = e^x$, $\frac{dy}{dx} = \ln e \cdot e^x = e^x$

Using the chain rule, for $y = e^{f(x)}$, $\frac{dy}{dx} = e^{f(x)}.f'(x)$.

e.g. $y = e^{\sin x} \Rightarrow \frac{dy}{dx} = \cos x \cdot e^{\sin x}$

Integration

$$\int f'(x)\cdot e^{f(x)}dx = e^{f(x)} + c$$

Logarithmic Functions

Differentiation

$$\begin{aligned}&y = \log_e x\\ &\Rightarrow e^y = x\\ &\frac{dx}{dy} = e^y\\ &\therefore \frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}\end{aligned}$$

If $y = \log_e x$, $\frac{dy}{dx} = \frac{1}{x}$

By the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dy}$

$$\frac{dy}{dx} = \frac{1}{f(x)} \times f'(x) = \frac{f'(x)}{f(x)}$$

If $\boldsymbol{y = \log_e f(x)}$, $\frac{dy}{dx} = \frac{1}{x}$