# Logarithms and Indices (Mathematics)

## Index Laws

$$a^m \times a^n = a^{m+n}$$
$$\frac{a^m}{a^n} = a^{m-n}$$
$$\left(a^m\right)^n = a^{mn}$$
$$(ab)^n = a^n b^m$$

## Zero and Negative Indices

$$a^{-n} = \frac{1}{a^n}$$
$$a^0 = 1$$

## Rational Indices

$a^{\frac{1}{q}}$, where $q$ is a positive integer and $a > 0$, is defined as the positive $q^{\text{th}}$ root of $a$.

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

If $a$ is negative and $q$ is an **even** integer, $a^{\frac{1}{q}}$ does not exist in the field of real numbers. If $a$ is negative and $q$ is an **odd** integer, $a^{\frac{1}{q}}$ does exist.

$$a^{\frac{p}{q}} = \left(\sqrt[q]{a}\right)^p$$
$$a^{-\frac{p}{q}} = \frac{1}{a^{\frac{p}{q}}}$$

## Logarithms

$$a^x = y \Leftrightarrow \log_a y = x$$

## Log Laws

$$\log_a 1 = 0 \ \text{(because } a^0 = 1\text{)}$$

$$\log_a a = 1 \ \text{(because } a^1 = a\text{)}$$

$$\log_a (mn) = \log_a m + \log_a n$$

$$\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$$

$$\log_a (m^p) = p \log_a m$$

$$\log_a \left(\frac{1}{n}\right) = -\log_a n$$

## Change of Base of Logarithms

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Since most scientific calculators use base 10, $b$ is usually 10.