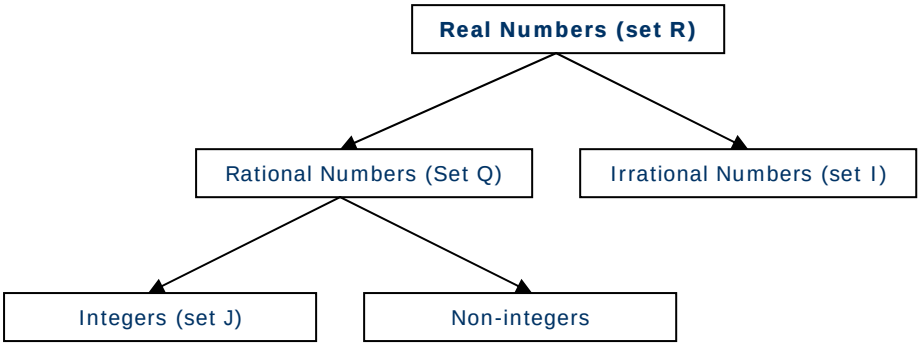


Basic Arithmetic and Algebra (Mathematics)

The Real Number System

The **Real number system** is denoted by the set **R**. It is divided into two sets of numbers: rational and irrational.



Rational numbers can be expressed in the form $\frac{a}{b}$, where a and b are integers, $b \neq 0$.

Examples

$\frac{3}{4}$ $\quad 40\% = \frac{2}{5}$ $\quad 0.\dot{3} = \frac{1}{3}$ $\quad 1{:}2 = \frac{1}{2}$ $\quad 2 = \frac{2}{1}$

Irrational numbers cannot be expressed in the above form. Only decimal approximations can be given for them.

Examples

π, e, $\sqrt{2}$, $\log_{10} 2$

Recurring Decimals

Example

Write $0.\dot{1}\dot{2}$ as a fraction.

Let $x = 0.\dot{1}\dot{2}$

Then,

$100x = 12.\dot{1}\dot{2} \Rightarrow 99x = 100x - x = 12$

Therefore $x = \frac{12}{99} = \frac{4}{33}$, i.e. $0.\dot{1}\dot{2} = \frac{4}{33}$.

Factorisation

Common Factor

Example

$(2x - y)^2 - 3(2x - y) = (2x - y)(2x - y - 3)$

Grouping in Pairs

Example

$$\begin{aligned} m^2 - mn - 2m + 2n &= m(m - n) - 2(m - n) \\ &= (m - 2)(m - n) \end{aligned}$$

Difference of two squares

$$a^2 - b^2 = (a - b)(a + b)$$

This theorem can also be used in reverse.

Quadratics

Example

$$\begin{aligned} 2x^2 - xy - 21y^2 &= 2x^2 + 6xy - 7xy - 21y^2 \\ &= 2x(x + 3y) - 7y(x + 3y) \\ &= (2x - 7)(x + 3y) \end{aligned}$$

Sum and Difference of two cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Surds

- Involve roots
- They are irrational, i.e. they are numbers that cannot be written in the form $\frac{a}{b}$, $b \neq 0$.

Simplifying Surds

Break the surd up into factors, one of which is a perfect square.

Example

$$\begin{aligned} 5\sqrt{48} &= 5\sqrt{16 \times 3} \\ &= 20\sqrt{3} \end{aligned}$$

Adding and Subtracting Surds

We must have LIKE terms; surds may need to be simplified first.

Example

$$\begin{aligned} \text{If } \sqrt{y} &= \sqrt{63} - \sqrt{7} + \sqrt{28} \text{ find } y. \\ \sqrt{y} &= 3\sqrt{7} - \sqrt{7} + 2\sqrt{7} \\ &= 4\sqrt{7} \\ &= \sqrt{112} \\ \therefore \quad y &= 112 \end{aligned}$$

Multiplication of Surds

Multiply numbers with numbers and the surds with surds.

Example

$$\left(5 - 2\sqrt{3}\right)\left(2\sqrt{5} - 6\right) = 10\sqrt{5} - 30 - 4\sqrt{15} + 12\sqrt{3}$$

$$\begin{aligned} \sqrt{a^2} &= a \\ \frac{1}{\sqrt{a}} &= \frac{\sqrt{a}}{a} \\ \sqrt{a} \cdot \sqrt{b} &= \sqrt{ab} \\ \frac{\sqrt{a}}{\sqrt{b}} &= \sqrt{\frac{a}{b}} \end{aligned}$$