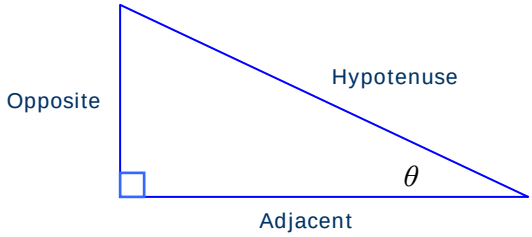


Trigonometry (Mathematics & Mathematics Extension 1)

The Trigonometric Ratios



In any right triangle, the trigonometric ratios are:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

SOH CAH TOA

The Reciprocal Ratios

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$$

Complementary Ratios

SIN and **COS** are complementary ratios.

$$\sin\theta = \cos(90° - \theta)$$
$$\cos\theta = \sin(90° - \theta)$$

TAN and **COT** are complementary ratios.

$$\tan\theta = \cot(90° - \theta)$$
$$\cot\theta = \tan(90° - \theta)$$

Ratios of Special Angles (Exact Values)

$$\sin 30° = \frac{1}{2} \qquad \cos 60° = \frac{1}{2} \qquad \cos 45° = \frac{1}{\sqrt{2}}$$
$$\cos 30° = \frac{\sqrt{3}}{2} \qquad \tan 60° = \sqrt{3} \qquad \tan 45° = 1$$
$$\tan 30° = \frac{1}{\sqrt{3}} \qquad \sin 45° = \frac{1}{\sqrt{2}}$$
$$\sin 60° = \frac{\sqrt{3}}{2}$$

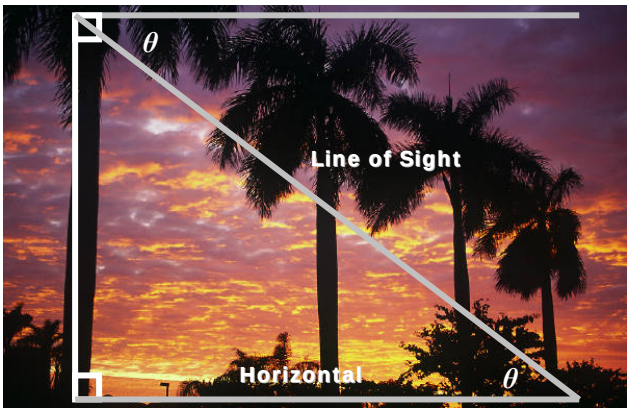
Bearings

True bearings are bearings from North. They are always **three digits**, e.g. 290°T.

Compass bearings are measured from the compass directions: North (N), South (S), East (E), West (W), e.g. N 70°E.

Angles of Elevation and Depression

- The **Angle of Elevation** is the angle between the line of sight and the horizontal, looking up at something.
- The **Angle of Depression** is the angle between the line of sight and the horizontal, looking down at something.



Angle of depression = Angle of elevation (by alternate angles on parallel lines)

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

IMPORTANT:

When dealing with the sine ratio, it is possible to have **acute** and **obtuse** solutions.

Cosine Rule

Finding **sides**:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + a^2 - 2ab\cos C$$

Finding **angles**:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Area of a triangle

$$A = \frac{1}{2}ab\sin C$$

Area = $\frac{1}{2}\times$ the product of the 2 sides $\times$ the sine of the included angle

Angles of any magnitude

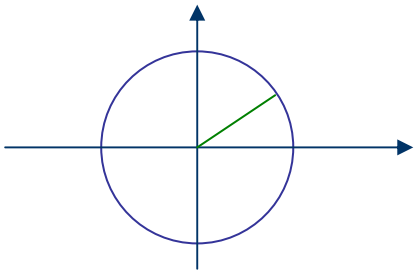
Positive angles are measured in an **anticlockwise direction**.
Negative angles are measured in a **clockwise direction**.

ASTC
All **S**tations **T**o **C**entral
for the signs of the ratios

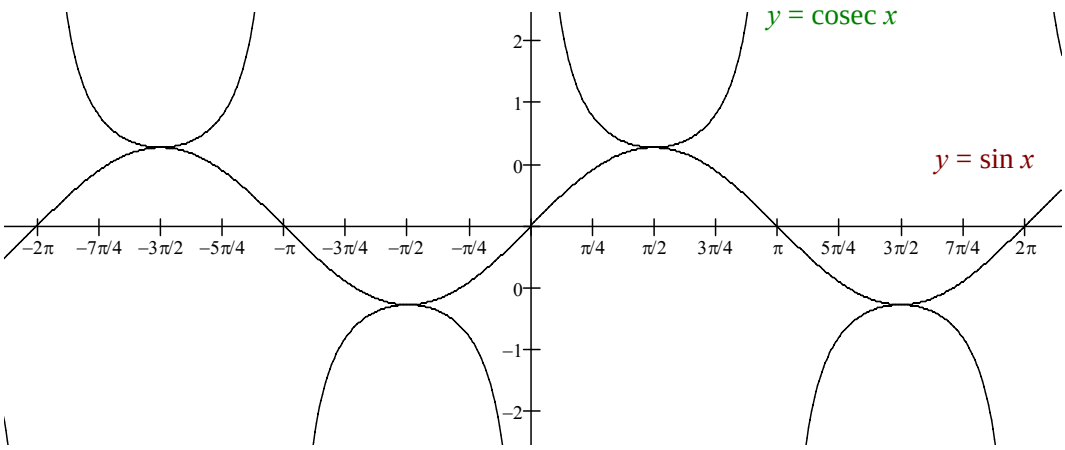
Ratios for Quadrant Boundary Angles

Examining a point (x, y) on the unit circle:

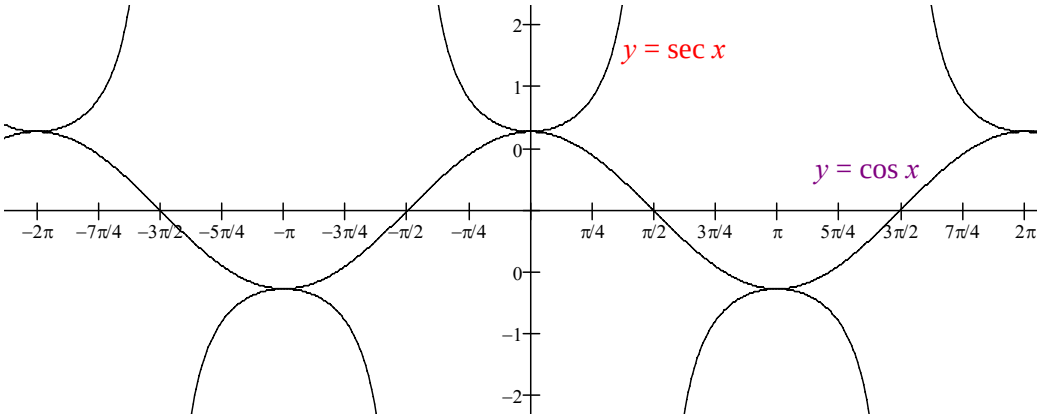
$\sin\theta = y$-coordinate
$\cos\theta = x$-coordinate
$\tan\theta = \dfrac{y\text{-coordinate}}{x\text{-coordinate}}$



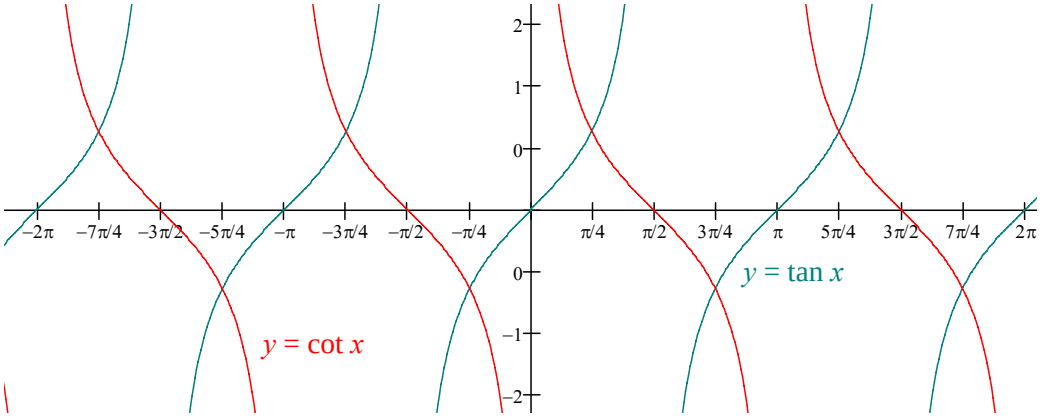
Graphs of the Trigonometric Ratios



	$y = \sin x$	$y = \text{cosec}\, x$
Domain	All real x	$-1 \le y \le 1$
Range	All reals except 0°, 180°, 360° etc.	$y \ge 1, y \le -1$
Period	360°	360°



	$y = \cos x$	$y = \sec x$
Domain	All real x	$-1 \le y \le 1$
Range	All reals except 90°, 270°, 450° etc.	$y \ge 1, y \le -1$
Period	360°	360°



	$y = \tan x$	$y = \cot x$
Domain	All reals except 90°, 270° etc.	All reals except 0°, 180°, 270° etc.
Range	All reals	All reals
Period	180°	180°

Trigonometric Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

Dividing all terms by **cos2 θ**,

$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 = \sec^2\theta - \tan^2\theta$$
$$\tan^2\theta = \sec^2\theta - 1$$

Dividing all terms by **sin2 θ**,

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$
$$1 = \operatorname{cosec}^2\theta - \cot^2\theta$$
$$\cot^2\theta = \operatorname{cosec}^2\theta - 1$$

3-Dimensional Trigonometry (Extension 1 only)

- A straight line is perpendicular to a plane if it is perpendicular to every straight line that lies in the plane and passes through its foot. The perpendicular is called the **normal** to the plane at that point.
- The angle between a line and a plane is the angle between the line and its projection on the plane.
- The angle between two planes is measured by the angle between two lines drawn from any point in the line of intersection at right angles to it, one to each plane.
- The angle between two planes is called the **dihedral angle**.
- The **line of greatest slope** is the line that is perpendicular to the horizon.