

Test on Circle Geometry (Chapter 15)

Chord Properties of Circles

A *chord* of a circle is any interval that joins two points on the curve.
The largest chord of a circle is its *diameter*.

1. Chords of equal length in a circle subtend equal angles at the centre.
2. Chords of equal length in a circle are equidistant from the centre.
3. The perpendicular from the centre of a circle to a chord bisects the chord.
4. The line from the centre of a circle to the midpoint of the chord is perpendicular to the chord.
5. The perpendicular bisector of a chord of a circle passes through the centre of the circle.
6. If two circles intersect, then the interval joining their centres is the perpendicular bisector of their common chord.

The circle that passes through the vertices of the triangle is called the *circumcircle* and the centre O is the *circumcentre*.

Angle properties of circles

1. The angle in a semicircle is a right angle.
2. Angles at the circumference standing on the same arc are equal.
3. A *cyclic quadrilateral* is a quadrilateral that has its vertices on the circumference of a circle (the points are called *concylic points*).

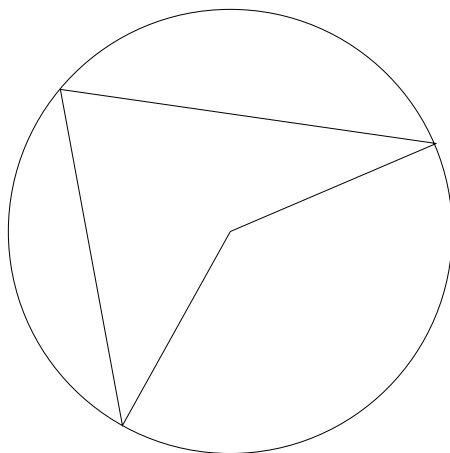
Tangents to a circle

A *tangent* to a circle is a line that touches the circle at only one point. The point of intersection is called the *point of contact*.

1. A tangent to a circle is perpendicular to the radius drawn to the point of contact.
2. The two tangents drawn to a circle from an external point are equal in length.
3. The angle between a tangent and a chord drawn to the point of contact is equal to the angle in the alternate segment.
4. When two circles touch, their centres and the point of contact are *collinear*.
5. The products of the intercepts of two intersecting chords are equal.
6. The products of the intercepts of two intersecting secants to a circle from an external point are equal.
7. The square of the tangent to a circle from an external point equals the product of the intercepts of any secant from the point.

Required proofs

F.3: The angle at the centre is twice an angle at the circumference subtended by the same arc.



Aim: To prove $\angle AOC = 2\angle ABC$

Construction: Join BO and produce to D

Proof:

Let $\angle OBA$ to be equal to a and $\angle OBC$ to be equal to β .

Now $\triangle AOB$ is isosceles ($OA = OB$, radii of circle)

$\angle OAB = a$ (base angles of isosceles \triangle)

$$\therefore \angle AOD = a + \beta$$

Similarly,

$$\angle COD = 2\beta$$

$$\angle AOC = 2a + 2\beta$$

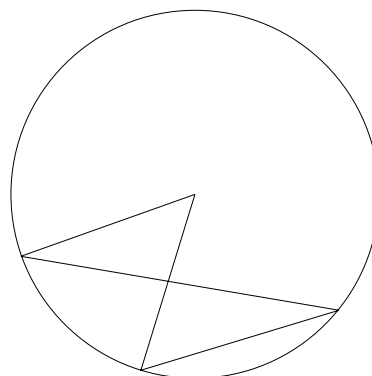
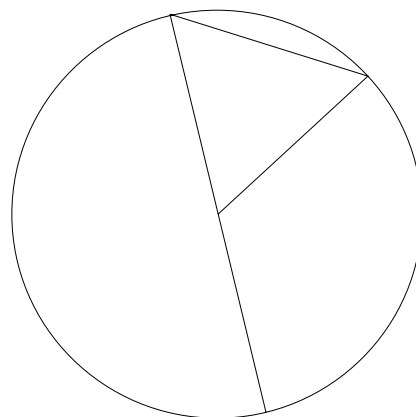
$$= 2(a + \beta)$$

Because $\angle ABC = a + \beta$ (adjacent angles),

$$\therefore \angle AOC = 2\angle ABC$$

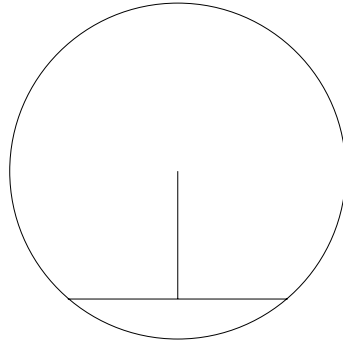
\therefore The angle at the centre is twice an angle at the circumference subtended by the same arc.

Other cases:



F.3: The perpendicular from the centre of a circle to a chord bisects the chord.

$$OM \perp AB \Rightarrow AM = MB$$



Given: $OM \perp AB$.

Aim: To prove $AM = MB$.

Proof:

In \triangle s AOM and BOM,

i) $OA = OB$ (radii of same circle)

ii) OM is common

iii) $\angle OMA = \angle OMB = 90^\circ$

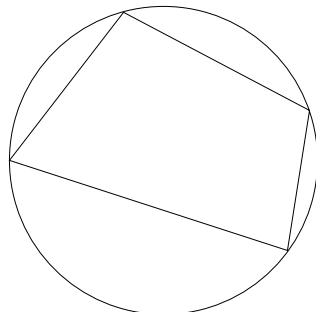
$\therefore \triangle AOM = \triangle BOM$ (RHS)

$\therefore AM = MB$ (corresponding sides of congruent triangles)

\therefore The perpendicular from the centre of a circle to a chord bisects the chord.

F.10: Opposite angles of a cyclic quadrilateral are supplementary.

$$x + y = 180^\circ, z + w = 180^\circ$$



Data: ABCD is any cyclic quadrilateral

Aim: To prove the opposite angles add up to 180°

Construction: Draw radii AO and OC

Proof:

Let $\angle ABC$ be α , $\angle ADC$ be β .

Now, reflex $\angle AOC = 2\beta$ (angle at centre is twice angle at circumference, standing on the same arc)

Also, obtuse $\angle AOC = 2\alpha$ (angle at centre is twice angle at circumference, standing on the same arc)

Now, $2\alpha + 2\beta = 360^\circ$ (angles at a point)

$$\alpha + \beta = 180^\circ$$

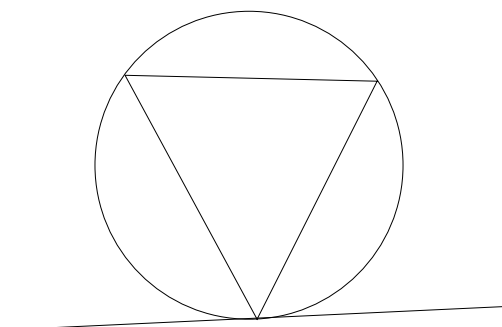
$$\therefore \angle ABC + \angle ADC = 180^\circ$$

Similarly, $\angle BAD + \angle BCD = 180^\circ$

\therefore Opposite angles of a cyclic quadrilateral are supplementary.

F.13: The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.

PTW is a tangent, $\angle BTW = \angle TAB$ and $\angle ATP = \angle ABT$.



Data: The chord BT meets the tangent PW at the point of contact T.
O is the centre of the circle

Aim: Prove $\angle BTW = \angle BAT$

Construction: Draw OT, OB

Proof:

Let $\angle BTW$ be x .

Now, $\angle OTW = 90$ (radius OT is perpendicular to tangent PW)

$$\angle OTB = 90 - x$$

$$\angle OBT = 90 - x$$

($\triangle TOB$ is isosceles, OT and OB are equal radii)

$$\begin{aligned} \therefore \angle TOB &= 180 - 2(90 - x) \\ &\text{(Angle sum of triangle)} \\ &= 2x \end{aligned}$$

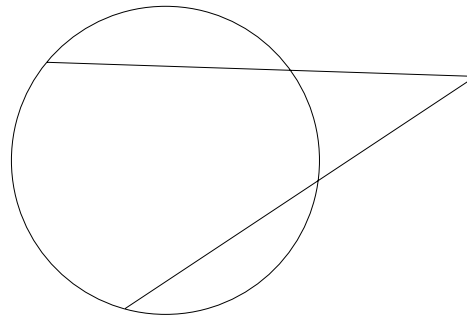
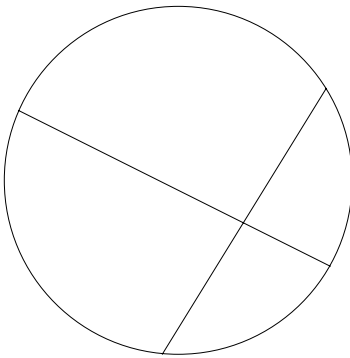
$\therefore \angle BTW = x$ (angle at the circumference is half the angle at the centre on the same arc)

$$\therefore \angle BTW = \angle BAT.$$

\therefore Angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.

F.16: If two chords AB, CD cut each other internally (or externally) at X, then $AX.XB = CX.XD$

Internal intersection



External intersection

INTERNAL:

Data: Chords AB and CD meet at X.

Aim: To prove that $AX.XB = CX.XD$

Construction: Draw AC, DB

Proof:

In \triangle s ACX and DBX:

i) $\angle ACX = \angle DBX$ (angles subtended by the same arc AD)

ii) $\angle AXC = \angle DXB$ (vertically opposite angles)

$\therefore \triangle ACX \sim \triangle DBX$ (equiangular)

$$\therefore \frac{AX}{XD} = \frac{CX}{XB} \text{ (Corresponding sides are in the same ratio)}$$

$$\therefore AX.XB = CX.XD \text{ (as required)}$$

EXTERNAL:

Data: Chords EF and GH meet at Y

Aim: To prove $EY \cdot YF = GY \cdot YH$

Construction: Draw FG and EH

Proof:

In \triangle s EYH and GYF:iii) $\angle EYH = \angle GYF$ (same angle)iv) $\angle YEH = \angle YGH$ (angles subtended by the same arc FH) $\therefore \triangle EYH \sim \triangle GYF$ (equiangular) $\therefore \frac{EY}{GY} = \frac{YH}{YF}$ (Corresponding sides are in the same ratio) $\therefore EY \cdot YF = GY \cdot YH$ (as required)