

# Surds (Chapter 1)

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## Rational and irrational numbers

- A **rational number** is any number that can be expressed in the form  $\frac{a}{b}$ , where  $b \neq 0$  and  $a$  and  $b$  are integers, whose highest common factor is 1. Examples of rational numbers include  $\frac{5}{6}$ , 24, -3,  $2\frac{5}{8}$ , 0.5, 8½% and 1.37.

An **irrational number** is a number that is not rational, so it cannot be expressed in the form  $\frac{a}{b}$ , where  $b \neq 0$  and  $a$  and  $b$  are integers. Examples of irrational numbers include  $\sqrt{2}$ ,  $-\sqrt{15}$  and  $\pi$ .

- An irrational number whose exact value can only be expressed using the radical or root symbol ( $\sqrt{\quad}$ ) is called a **surd**. Examples of surds include  $\sqrt{2}$ ,  $-\sqrt{5}$ ,  $\sqrt[3]{4}$  and  $\sqrt[5]{8}$ . The exact value of the surd is the surd itself.  $\pi$  is an irrational number, but not a surd.

## Simplifying Surds

For  $a, b > 0$

- $(\sqrt{a})^2 = \sqrt{a^2} = a$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

A surd is simplified when no more perfect squares (or perfect cubes, etc.) can be found in the factors, eg the simplest form of  $\sqrt{98}$  is  $7\sqrt{2}$  since 2 has no factors which are perfect squares.

Simplified surds can be written as entire surds by reversing the simplifying process.

## Multiplying and dividing surds

For  $a, b > 0$ ,

- $c\sqrt{a} \times d\sqrt{b} = cd\sqrt{ab}$
- $\frac{c\sqrt{a}}{d\sqrt{b}} = \frac{c}{d}\sqrt{\frac{a}{b}}$

## Adding and subtracting surds

When simplifying expressions involving the addition and subtraction of surds, add and subtract the terms that contain the same surd. You may first need to express all the surds in their simplest form.

## Rationalising the denominator

- $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

$$2. \frac{a\sqrt{d}}{c\sqrt{b}} = \frac{a\sqrt{d}}{c\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{db}}{cb}$$

### Rationalising surds with binomial denominators

$$1. \frac{a}{\sqrt{b} + \sqrt{c}} = \frac{a}{\sqrt{b} + \sqrt{c}} \times \frac{\sqrt{b} - \sqrt{c}}{\sqrt{b} - \sqrt{c}} = \frac{a(\sqrt{b} - \sqrt{c})}{b - c}$$
$$2. \frac{a}{\sqrt{b} - \sqrt{c}} = \frac{a}{\sqrt{b} - \sqrt{c}} \times \frac{\sqrt{b} + \sqrt{c}}{\sqrt{b} + \sqrt{c}} = \frac{a(\sqrt{b} + \sqrt{c})}{b - c}$$

### Conjugate surds

The expressions  $(\sqrt{b} + \sqrt{c})$  and  $(\sqrt{b} - \sqrt{c})$  are said to be **conjugate surds**. The product of conjugate surds is always a rational number.

## Rates and variation (Chapter 2)

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### Rates

A **ratio** compares quantities of the same kind but a **rate** compares quantities of different kinds.

Note: A **knot** is a speed of 1 nautical mile (1852 metres) per hour.

### Variation

**Variation** is the relationship between two quantities such that a change in one causes a change in the other.

Variation may be **direct** or **inverse**.

When two variables change in a way that their ratio remains fixed, it is called **direct variation**. Thus, the circumference of a circle *varies directly* with its diameter ( $\pi$  is called the **constant of variation** in  $C = \pi d$ ).

When two variables change in a way that their product remains fixed, it is called **inverse variation**. Thus, if  $xy = 12$ ,  $x$  varies inversely with  $y$ .

### Direct variation

Direct variation occurs when a change in one quantity directly affects the other.

Distance varies directly with time

$D$  varies directly with  $T$

$D \propto T$

$D = kT$

(in words)

(using symbols)

( $\propto$  means *varies* or *is proportional to*)

( $k$  is the constant of variation)

Each of the equations using squaring, cubing... and using square roots, cube roots... are examples of **non-linear direct variation**. The change in one variable still directly affects the change in the other, but the equations have non-linear graphs.

### Finding the variation equation

x	0	1	2	3	4
y	0	3	12	27	48

1<sup>st</sup> difference

3      9      15      21

2<sup>nd</sup> difference

6      6      6

Constant pattern is in the 2<sup>nd</sup> difference, so y is related to x squared, ie. the equation is of the form  $y = kx^2$ .

### Inverse variation

For **inverse variation**, when one variable increases the other decreases:

Time taken varies inversely with speed

(in words)

T varies inversely with S

(using symbols)

$$T \propto \frac{1}{S}$$

( $\propto$  means *varies* or *is proportional to*)

$$T = \frac{k}{S}$$

(k is called the constant of variation)

## Surface Area and Volume (Chapter 3)

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### Surface area

The surface area of a solid is the sum of the areas of all its faces. To calculate the surface area of a solid, find the area of each face and then add the areas together.

### Volume and capacity

**Volume** is the amount of space an object occupies. **Capacity** measures the maximum amount the object can contain.

### Surface area of a pyramid

The surface area of a pyramid is calculated by adding the area of the base and the area of the triangular faces.

The **slant height** of a right pyramid is the distance from the vertex to the base along a triangular face. For a cone, the slant height is the distance from the vertex to its circular base.

### Surface area of a cone

Surface area of a sphere

Volume of a pyramid

Volume of a cone

Volume of a sphere

Area and volume of similar figures and solids

Similar figures or solids have the same shape, their corresponding angles are equal, and the

For a sphere,

$$\text{Surface area} = 4\pi r^2$$

Where  $r$  = radius of sphere

length of their corresponding sides are in the same ratio.

If the corresponding sides of two similar figures are in the ratio  $m : n$  then:

- The ratio of their surface areas is  $m^2 : n^2$
- The ratio of their volumes is  $m^3 : n^3$

The volume of a pyramid is given by  $V = \frac{1}{3} Ah$

Where  $A$  is the area of the base and  $h$  is the height of the pyramid

The volume of a cone is given by

$$V = \frac{1}{3} \pi r^2 h$$

Where  $r$  is the radius of the base and  $h$  is the height of the cone

The volume of a sphere is given by  $V = \frac{4}{3} \pi r^3$

Where  $r$  is the radius of the sphere

# Consumer arithmetic (Chapter 4)

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## Bonuses, allowances and incentives

Employees sometimes receive the following:

- A **bonus** is an amount of money paid to an employee over and above his/her normal pay.
- An **allowance** is an amount of money paid to an employee to meet expenses. Examples include travel, meal and wet weather allowances.
- An **incentive** is an amount of money or a benefit given to employees to encourage an increase in effort that may result in greater output or an improvement in quality.

## Simple interest

The amount of money that is invested (or borrowed) is usually referred to as the **principal**.

**Simple interest** is interest paid on the original amount invested or borrowed (the principal). It is also referred to as the **flat rate** of interest.

Simple interest can be calculated by:

$$I = Prt$$

Where:

I = simple interest earned (or charged)

P = principal (amount invested or borrowed)