Polynomials (Mathematics Extension 1)

Remainder and Factor Theorem

- If $P(x) \div (x-a)$, the remainder is equal to $P(a)$
- If $(x-a)$ is a factor of P(x), then P(a) = 0

Relationships Between the Roots

Quadratics

If α, β are the roots of $y = ax^2 + bx + c$

- $\alpha + \beta = -\frac{b}{a}$
- $\alpha\beta = \frac{c}{a}$

Cubics

If α, β, γ are the roots of $y = ax^3 + bx^2 + cx + d$

- $\Sigma\alpha = \alpha + \beta + \gamma = -\frac{b}{a}$
- $\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
- $\Sigma\alpha\beta\gamma = \Pi\alpha = \alpha\beta\gamma = -\frac{d}{a}$

Quartics

If $\alpha, \beta, \gamma, \delta$ are the roots of $y = ax^4 + bx^3 + cx^2 + dx + e$

- $\Sigma\alpha = -\frac{b}{a}$
- $\Sigma\alpha\beta = \frac{c}{a}$
- $\Sigma\alpha\beta\gamma = -\frac{d}{a}$
- $\Sigma\alpha\beta\lambda\delta = \frac{e}{a}$

Approximation of Roots

Halving the Interval

For a continuous function, there must be at least one root (i.e. it crosses the x-axis) in the interval between two values that have opposite signs (say x_1 and x_2).
The first approximation to the root is the average of x_1 and x_2.

Newton's Method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

x_1 = first estimate
x_2 = next estimate