MATH3964 Lecture Week 12-1

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1 Gauss' formula for $F(\alpha, \beta; \gamma; 1)$

- The series $F(\alpha, \beta; \gamma; 1) = \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{(\gamma)_n n!}$ converges absolutely whenever Re $(\gamma - \alpha - \beta) > 0$ or when the series terminates. In all other cases, $F(\alpha, \beta; \gamma; z)$ does not analytically continue to $z = 1$.

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$$\begin{aligned} F(\alpha, \beta; \gamma; z) &= \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{(\gamma)_n n!} z^n \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{(\beta)_n}{(\gamma)_n} \binom{-\alpha}{n} z^n \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{\Gamma(\gamma)\Gamma(\beta+n)}{\Gamma(\beta)\Gamma(\gamma+n)} \binom{-\alpha}{n} z^n \end{aligned}$$

  Continued in OneNote - integral formula

- (Using binomial coefficient $\binom{-\alpha}{n} = (-1)^n \frac{(\alpha)_n}{n!}$)